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Electromagnetic Radiation Influence on the Transversal Conductivity of the Graphene Superlattice

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The transversal conductivity of the graphene superlattice under electromagnetic radiation and constant electric field applied along the superlattice axis was calculated. The cases of sinusoidal and cnoidal electromagnetic waves are considered. Conductivity dependences on the electromagnetic wave amplitude and on the longitudinal electric field intensity were investigated. Such dependences were shown to have the character of oscillations.

Keywords: Graphene, Superlattice, Cnoidal wave.

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1. INTRODUCUCTION

The unusual electron spectrum of graphene-based structures leads to remarkable electronic and optical properties which were intensively studied experimentally and theoretically [1-12]. Such investigations are of fundamental and practical interest [13-18]. The quantum theory of the magneto-optical conductivity was developed in [19, 20]. A quasi-classical kinetic theory of the nonlinear EM response of graphene was developed in [21-24]. Also in [22-24] the possible applications of the predicted effects for generation of terahertz (THz) radiation were discussed. The mutual rectification of the EM waves in graphene were studied in [25-27].

The predicted nonlinear optical properties of the graphene-based structures open new opportunities for building of the optoelectronic devices. The band structure of the graphene superlattice (GSL) was studied in [28-36]. Intensive investigations of the electronic and optical features of graphene structures with additional SL potential are of interest due to the so-called Bloch oscillator problem [37, 38]. Moreover SL is the suitable medium for the formation of the nonlinear and solitary EM waves [39-43].

The optical properties of GSL were investigated in [34, 35, 44], where the constant electric current induced in GSL by the EM waves of different polarization was calculated. In [35, 44] one of the waves was of cnoidal form. The negative differential conductivity and absolute negative conductivity of the GSL axis was shown to be possible under the simultaneous action of EM wave polarized elliptically and constant electric field applied along the GSL axis.

In experiment, however, it is very difficult to orient the electric field vector along the GSL axis strictly. There always exists a small transverse component of the of applied electric field intensity.

Thus, it was of interest to calculate the transversal conductivity of the GSL in the presence of a weak transverse electric field in addition to a strong constant electric field applied along the GSL axis and EM radiation. The dependence of the transversal conductivity on the longitudinal electric field intensity is shown below to have a resonant character.

2. TRANSVERSAL CONDUCTIVITY OF GSL UNDER THE SINUSOIDAL EM WAVE

GSL is considered to be obtained by a sheet of graphene deposited on a banded substrate formed by periodically alternating layers of SiO_2 and SiC. The layers are arranged so that the hexagonal crystal lattice of SiC was under the hexagonal lattice of graphene.

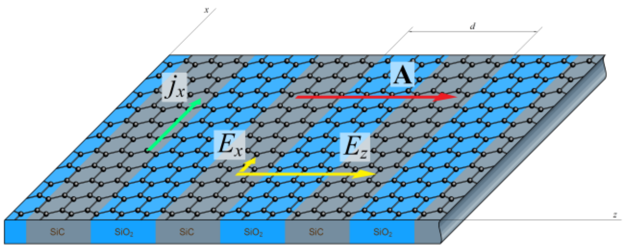

Fig. 1 – Schematic of the process

Due to this, in the areas of graphene plane located above the layers of SiC an energy gap $2\Delta = 0.23$ eV arises [2, 34]. Let the graphene is located on the plane xz. The electron spectrum of GSL has the approximate view [34, 35]:

$$\varepsilon(\mathbf{p}) = \sqrt{\varepsilon_0^2 + p_x^2 \upsilon_F^2} + \frac{\varepsilon_1^2}{\sqrt{\varepsilon_0^2 + p_x^2 \upsilon_F^2}}\left(1 - \cos\frac{p_z d}{\hbar}\right), \quad (1)$$

where $\varepsilon_0 = 0.059$ eV, $\varepsilon_1 = 0.029$ eV, $d = 2 \cdot 10^{-6}$ cm is the SL period, $\upsilon_F = 10^8$ cm/s is the Fermi surface velosity, Oz is the SL axis. With the new notations:

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$q_x = p_x d/\hbar$, $q_z = p_z d/\hbar$, $\gamma = \hbar\upsilon_F/d\varepsilon_0$, $\gamma_1 = \varepsilon_1/\varepsilon_0$, the expression (1) takes the next form:

$$\varepsilon(\mathbf{p}) = \varepsilon_0\sqrt{1+\gamma^2 q_x^2} + \frac{\varepsilon_0\gamma_1^2(1-\cos q_z)}{\sqrt{1+\gamma^2 q_x^2}}. \quad (2)$$

The polarization plane of the sinusoidal EM wave is considered to be perpendicular to the GSL plane so that its electric field intensity is applied along the SL axis (Fig. 1). The vector potential of the EM wave is

$$\mathbf{A}(t) = \left(0,\, 0,\, \frac{cE_0}{\omega}\sin\omega t\right), \quad (3)$$

where E_0 and ω are the amplitude and the frequency of the EM wave correspondingly. The constsnt electric field is $\mathbf{E} = (E_x\,, 0\,, E_z)$. Under these conditions the electric current density arising through the axis Ox in the constant relaxation time τ approximation is calculated with the following formula:

$$j_x(t) = -\frac{e}{\tau}\int_{-\infty}^{t} dt' e^{-\frac{t-t'}{\tau}} \sum_{\mathbf{p}} f_0(\mathbf{p}) \times \\ \times V_x\left(\mathbf{p} + \frac{e}{c}\left(\mathbf{A}(t) - \mathbf{A}(t')\right) - e\mathbf{E}(t-t')\right), \quad (4)$$

where $f_0(\mathbf{p})$ is equilibrium state function, $V_x = \partial\varepsilon/\partial p_x$ is the electron velocity along the axis Ox. We rewrite (4) as

$$j_x(t) = -\frac{e\upsilon_F\gamma}{\tau}\int_{-\infty}^{t} dt' e^{-\frac{t-t'}{\tau}} \sum_{\mathbf{p}} f_0(\mathbf{p}) \\ \left(\frac{q_x+\beta'-\beta+\gamma^2(q_x+\beta'-\beta)^3}{\left(1+\gamma^2(q_x+\beta'-\beta)^2\right)^{3/2}} + \right. \\ \left. + \frac{\gamma_1^2(q_x+\beta'-\beta)\cos\left(q_z+\omega_B(t'-t)+\alpha'-\alpha\right)}{\left(1+\gamma^2(q_x+\beta'-\beta)^2\right)^{3/2}}\right), \quad (5)$$

where $\omega_B = eE_z d/\hbar$ is frequency of Bloch oscillations, $\alpha(t) = edA(t)/c\hbar$, $\alpha' = \alpha(t')$, $b = edE_x\tau/\hbar$, $\beta(t) = bt/\tau$. Further the transversal field is considered to be weak: $b \ll 1$. Taking into account the evenness of the equilibrium distribution function, we find that, in the second approximation with respect to the parameter b, formula (5) takes the form:

$$j_x(t) = -\frac{e\upsilon_F\gamma}{\tau}\int_{-\infty}^{t} dt' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\sum_{\mathbf{p}} f_0(\mathbf{p})\frac{1}{\left(1+\gamma^2 q_x^2\right)^{3/2}} - \\ -\frac{e\upsilon_F\gamma_1^2\gamma}{\tau}\int_{-\infty}^{t} dt' e^{-\frac{t-t'}{\tau}}(\beta'-\beta)\cos\left(\omega_B(t'-t)+\alpha'-\alpha\right)\times \\ \times\sum_{\mathbf{p}} f_0(\mathbf{p})\frac{\left(1-2\gamma^2 q_x^2\right)\cos q_z}{\left(1+\gamma^2 q_x^2\right)^{5/2}} . \quad (6)$$

The electrons are suggested to obey the Boltzmann statistics in the absence of the field. At low temperatures $\theta \ll \varepsilon_0$ we obtain

$$\langle j_x\rangle = \sigma_\perp E_x\left(1+\gamma_1^2\sum_{n=-\infty}^{\infty} J_n^2(a)\frac{1-(\omega_B+n\omega)^2\tau^2}{\left(1+(\omega_B+n\omega)^2\tau^2\right)^2}\right), \quad (7)$$

where $a = eE_0 d/\hbar\omega$, $\sigma_\perp = n_0 e^2\upsilon_F^2\tau/\varepsilon_0$ is the GSL conductivity in the direction perpendicular to the GSL axis in the absence of electric fields applied along the GSL axis, n_0 is the surface concentration of free electrons of the GSL. $J_n(x)$ is the Bessel function of whole order.

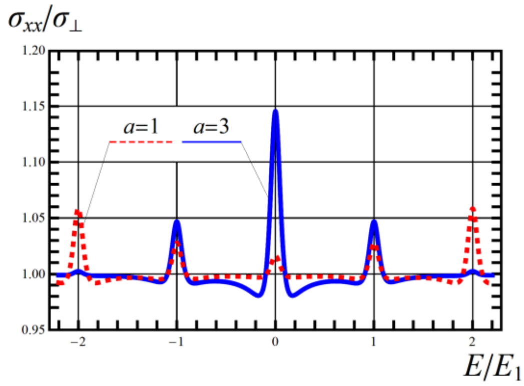

Fig. 2 – GSL transverse conductivity dependence on the longitudinal electric field intensity

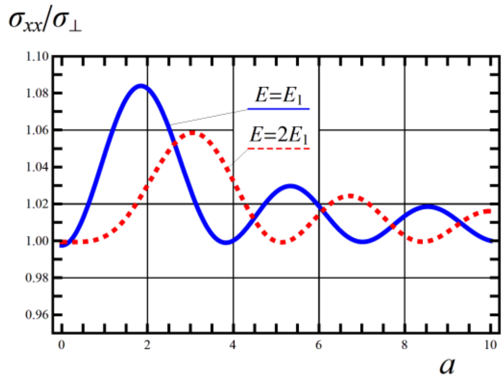

Fig. 3 – GSL transverse conductivity dependence on the EM wave amplitude

The transversal conductivity of GSL is

$$\sigma_{xx} = \sigma_\perp\left(1+\gamma_1^2 J_0^2(a)\frac{1-\omega_B^2\tau^2}{\left(1+\omega_B^2\tau^2\right)^2} + \gamma_1^2\sum_{n=1}^{\infty} J_n^2(a)\times \right. \\ \left. \times\left(\frac{1-(\omega_B+n\omega)^2\tau^2}{\left(1+(\omega_B+n\omega)^2\tau^2\right)^2} + \frac{1-(\omega_B-n\omega)^2\tau^2}{\left(1+(\omega_B-n\omega)^2\tau^2\right)^2}\right)\right). \quad (8)$$

Graphs of dependences of the GSL transverse conductivity on the longitudinal electric field intensity and on the EM wave amplitude are shown in Fig. 2 and 3 correspondingly $(E_1 = \hbar\omega/ed)$.

3. TRANSVERSAL CONDUCTIVITY OF GSL UNDER THE CNOIDAL EM WAVE

If the EM wave propagates in direction perpendicular to the GSL axis so that polarization plane of this wave coincides with the GSL plane then it has the form of cnoidal wave [35, 43, 48]. Now we calculate the transversal conductivity of the GSL exposed to the cnoidal EM wave. For this purpose we substitute the following Fourier series expansion in the formula (6):

$$e^{i\alpha(t)} = \sum_{n=-\infty}^{\infty} c_n(\kappa) e^{in\Omega(\kappa)t}, \tag{9}$$

where

$$\Omega(\kappa) = \frac{\pi\Omega_0}{2\mathrm{K}(\kappa)},\ c_0 = \frac{2\mathrm{E}(\kappa)}{\mathrm{K}(\kappa)} - 1,$$

$$c_n = \frac{n\pi^2}{\mathrm{K}^2(\kappa)} \frac{1}{q_\kappa^{-n/2} + (-1)^{n+1} q_\kappa^{n/2}}$$

$$\text{if } 0 < \kappa \le 1,$$

$$\Omega(\kappa) = \frac{\pi\Omega_0\kappa}{\mathrm{K}(\kappa^{-1})},\ c_0 = 1 - 2\kappa^2 + \frac{2\kappa^2\mathrm{E}(\kappa^{-1})}{\mathrm{K}(\kappa^{-1})},$$

$$c_n = \frac{4n\pi^2\kappa^2}{\mathrm{K}^2(\kappa^{-1})} \frac{1}{q_{1/\kappa}^{-n} - q_{1/\kappa}^{3n}},\ \text{if } \kappa > 1,$$

$$\kappa = \frac{eE_0 d}{2\hbar\Omega_0},\ q_\kappa = \exp\left(-\frac{\pi\mathrm{K}\left(\sqrt{1-\kappa^2}\right)}{\mathrm{K}(\kappa)}\right),$$

$$\Omega_0 = \frac{\omega_{\mathrm{pl}} u}{c}\sqrt{\left|1 - \frac{u^2}{c^2}\right|},$$

$$\omega_{\mathrm{pl}} = \frac{2ed\varepsilon_1}{\hbar}\sqrt{\frac{\pi n_0}{\varepsilon_0}},$$

E_0 is the amplitude of cnoidal wave, u is the wave velocity, c is the wave velocity in the absence of electrons, $\mathrm{K}(x)$ и $\mathrm{E}(x)$ are complete elliptic integrals of the first and second kind correspondingly. After substituting (9) in (6), calculation of integrals, and averaging over the wave period, we obtain the next formula for the GSL transversal conductivity at low temperatures $(\theta << \varepsilon_0)$:

$$\sigma_{xx} = \sigma_\perp \left(1 + \gamma_1^2 c_0^2 \frac{1-\omega_B^2\tau^2}{\left(1+\omega_B^2\tau^2\right)^2} + \gamma_1^2 \sum_{n=1}^{\infty}\left(\frac{c_n^2\left(1-(\omega_B+n\Omega)^2\tau^2\right)}{\left(1+(\omega_B+n\Omega)^2\tau^2\right)^2} + \frac{c_{-n}^2\left(1-(\omega_B-n\Omega)^2\tau^2\right)}{\left(1+(\omega_B-n\Omega)^2\tau^2\right)^2}\right)\right), \tag{10}$$

In the case $0 < \kappa \le 1$ we obtain

$$\sigma_{xx} = \sigma_\perp \left(1 + \gamma_1^2 c_0^2 \frac{1-\omega_B^2\tau^2}{\left(1+\omega_B^2\tau^2\right)^2} + \gamma_1^2 \sum_{n=1}^{\infty} c_n^2 \left(\frac{1-(\omega_B+n\Omega)^2\tau^2}{\left(1+(\omega_B+n\Omega)^2\tau^2\right)^2} + \frac{1-(\omega_B-n\Omega)^2\tau^2}{\left(1+(\omega_B-n\Omega)^2\tau^2\right)^2}\right)\right). \tag{11}$$

Graphs of dependences of the GSL transverse conductivity on the longitudinal electric field intensity plotted with the formula (11) are shown in Fig. 4 $(E_2 = \hbar\Omega_0/ed)$.

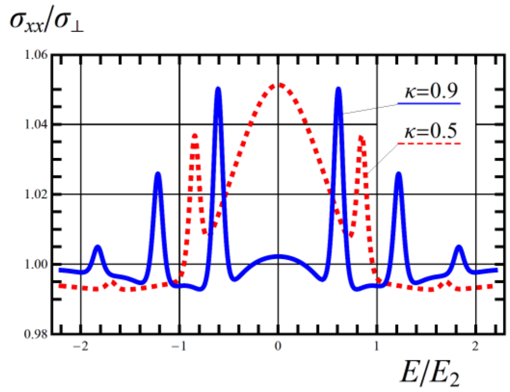

Fig. 4 – GSL transverse conductivity dependence on the longitudinal electric field intensity

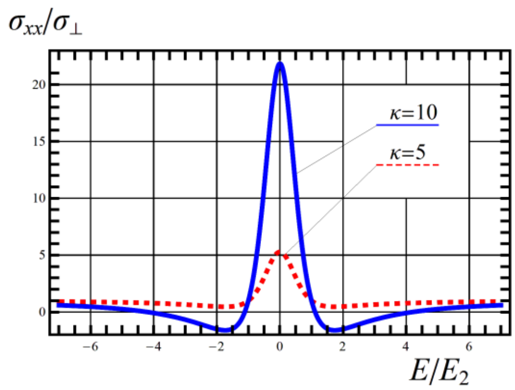

Fig. 5 – GSL transverse conductivity dependence on the longitudinal electric field intensity

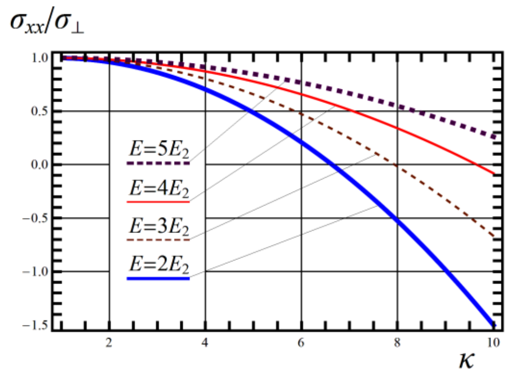

Fig. 6 – GSL transverse conductivity dependence on the cnoidal wave amplitude

In the case $\kappa > 1$ we have

$$\sigma_{xx} = \sigma_{\perp}\left(1 + \gamma_1^2 c_0^2 \frac{1 - \omega_B^2 \tau^2}{\left(1 + \omega_B^2 \tau^2\right)^2} + \right.$$
$$+\gamma_1^2 \sum_{n=1}^{\infty} c_n^2 \left(\frac{1 - \left(\omega_B + n\Omega\right)^2 \tau^2}{\left(1 + \left(\omega_B + n\Omega\right)^2 \tau^2\right)^2} + . \right. \quad (12)$$
$$\left.\left. + \frac{1 - \left(\omega_B - n\Omega\right)^2 \tau^2}{\left(1 + \left(\omega_B - n\Omega\right)^2 \tau^2\right)^2} q_{1/\kappa}^{4n} \right)\right)$$

Graphs of dependences of the GSL transverse conductivity on the longitudinal electric field intensity and on the EM wave amplitude plotted with the formula (12) are shown in Fig. 5 and 6.

4. DISCUSSIONS

The dependence of σ_{xx} on the longitudinal electric field intensity is seen from Fig. 2 to have the view of the alternating resonances. Conductivity peaks occur when the next condition is performed:

$$\omega_B = n\omega,$$

where n is the whole number. The dependence of σ_{xx} on the wave amplitude is seen from Fig. 3 to have the character of oscillations.

As can be seen from the Fig. 5 and 6, in the case of high intensities of cnoidal waves the so-called states of absolute negative conductivity are possible. There are the values of the cnoidal wave amplitude and of the longitudinal electric field when the transverse conductivity changes sign.

The possibility of the direct current in the transverse direction with relation to the SL axis is explained by the non-additivity of the GSL spectrum (1). In the bulk SL with additive spectrum such effect is impossible.

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